Introduction to Constraint Programming

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Constraint Programming

- Formulate the model using a variety of constraint types.
- Find the feasible solutions that satisfy all constraints.
- Search for the optimal solution.
Example: Suppose that we have the four variables, \( x_1, x_2, x_3, x_4 \), with their domains

\[
\begin{align*}
  x_1 &\in \{1,2\} \\
  x_2 &\in \{1,2\} \\
  x_3 &\in \{1,2,3\} \\
  x_4 &\in \{1,2,3,4,5\}
\end{align*}
\]
We also have the following constraints.

\[ x_i \neq x_j, \quad i \neq j \]

\[ x_1 + x_3 = 4 \]
Domain Reduction & Constraint Propagation

Since \( x_1 \in \{1,2\} \) and \( x_2 \in \{1,2\} \), the first constraint \( x_i \neq x_j, \ i \neq j \) implies that

\[ x_3 \in \{3\} \]

It then implies again

\[ x_4 \in \{4,5\} \]
We can then write,

\[ x_1 \in \{1\} \]
\[ x_2 \in \{2\} \]
\[ x_3 \in \{3\} \]
\[ x_4 \in \{4,5\} \]
Example Constraints

The “All-Different” Constraint

\[ \text{all-different} \left( y_1, y_2, \ldots, y_n \right) \]

The “Element” Constraint

\[ \text{element} \left( y, [c_1, c_2, \ldots, c_n], z \right) \]
Assignment Problem

\[
\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

\[
\sum_{i=1}^{n} x_{ij} = 1, \quad \forall j
\]

\[
\sum_{j=1}^{n} x_{ij} = 1, \quad \forall i
\]

\[
x_{ij} \in \{0,1\}
\]
Assignment Problem

If we define $y_i$ as the task assigned to person $i$, we can write

$$\min z = \sum_{i=1}^{n} z_i$$

element $(y_i, [c_{i1}, c_{i2}, \ldots, c_{in}], z_i)$, $i = 1, \ldots, n$

all-different $(y_1, y_2, \ldots, y_n)$

$y_i \in \{1, \ldots, n\}$, $i = 1, \ldots, n$
using CP;

int nbPerm = ...;
range r = 1..nbPerm;
int dist[r][r] = ...;
int flow[r][r] = ...;

execute{
    cp.param.timeLimit=30;
}

dvar int perm[1..nbPerm] in r;

dexpr int cost[i in r][j in r] =
    dist[i][j]*flow[perm[i]][perm[j]];

minimize sum(i in r, j in r) cost[i][j];
subject to {
    allDifferent(perm);
};
The End

Questions?