END5504: Non-Linear Programming

Example 1 (Advertising Selection):

Recall that the General Flakes Company from Example 4.1 of Chapter 4 sells a brand of low-fat breakfast cereal that appeals to people of all age groups and both genders. The company has advertised this product in various media for a number of years and has accumulated data on its advertising effectiveness. For example, the company has tracked the number of exposures to young men from ads placed on a particular television show for five different time periods. In each of these time periods, a different number of ads was used. Specifically, the numbers of ads were 1, 8, 20, 50, and 100. The corresponding numbers of exposures (in millions) were 4.7, 22.1, 48.7, 90.3, and 130.5. What type of nonlinear response function might fit these data well? Use the following function to predict exposures:

$$f(x) = a(1 - e^{-bx})$$

Example 2 (Advertising Selection-continued):

In this example, we revisit the problem faced by the General Flakes Company. The company must decide how many ads to place on each of several television shows to meet exposure constraints for each of six groups of customers. The difference now is that each combination of television show and customer group has its own advertising response function. That is, there are constants a and b of the response function for each such combination. (These constants appear in rows 5 to 10 and 14 to 19 of the file.) The company wants to find the selection of ads that minimizes its total cost of meeting all exposure requirements. We use a nonlinear model to find a minimum-cost way of meeting all exposure requirements.

Example 3 (Facility Location)

The Lafferty Company wants to locate a warehouse from which it will ship products to four customers. The location (in the x-y plane) of the four customers and the number of shipments per year needed by each customer are given in Table 3. (All coordinates are in miles, relative to the point $x = 0$ and $y = 0$) A single warehouse must be used to service all of the customers. Lafferty wants to determine the location of the warehouse that minimizes the total distance traveled from the warehouse to the customers.

Customer	x-coordinate	y-coordinate	# of shipments
C1	5	10	200
C2	10	5	150
C3	0	12	200
C4	12	0	300

Table 1: Problem Data

Example 4 (Single Variable)

Maximize the following function where $1 \leq x \leq 5$:

$$f(x) = \sum_{i=1}^{5} (x - i)$$

Example 5 (Portfolio Optimization)

I have \$1,000 to invest in three stocks. Let S_i be the random variable representing the annual return on \$1 invested in stock i. Thus, if $S_i = 0.12$, \$1 invested in stock i at the beginning of a year was worth \$1.12 at the end of the year. We are given the following information:

$$E(S_1) = 0.14, E(S_2) = 0.11, E(S_3) = 0.10$$

$$\text{var}(S_1) = 0.20, \text{var}(S_2) = 0.08, \text{var}(S_3) = 0.18$$

$$\text{cov}(S_1, S_2) = 0.18, \text{cov}(S_1, S_3) = 0.02, \text{cov}(S_2, S_3) = 0.03$$

Formulate a QPP that can be used to find the portfolio that attains an expected annual return of at least 12% and minimizes the variance of the annual dollar return on the portfolio.