END5504: Network Models

Example 1 (Transportation Model):

The Grand Prix Automobile Company manufactures automobiles in three plants and then ships them to four regions of the country. The plants can supply the amounts listed in the right column of Table 1. The customer demands by region are listed in the bottom row of this table, and the unit costs of shipping an automobile from each plant to each region are listed in the middle of the table. Grand Prix wants to find the lowest-cost shipping plan for meeting the demands of the four regions without exceeding the capacities of the plants.

	Region 1	Region 2	Region 3	Region 4	Capacity
Plant 1	131	218	266	120	450
Plant 2	250	116	263	278	600
Plant 3	178	132	122	180	500
Demand	450	200	300	300	

Table 1: Problem Data

How should you distribute the automobiles from plants to regions to minimize the transportation cost?

$$\min \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

$$\sum_{j=1}^{4} x_{ij} = s_i, \quad \forall i$$

$$\sum_{i=1}^{3} x_{ij} = d_j, \quad \forall j$$

$$x_{ij} \in \mathbb{Z}, \quad \forall i, j$$

$$x_{ij} \geq 0, \quad \forall i, j$$

What happens if $\sum_i s_i \neq \sum_j d_j$?

Model Definitions:

- Input Variables
    - plant capacities
    - regional demands
    - unit shipping costs
- Decision Variables
    - # of automobiles for each plants-region pair
- Objective Function
    - total shipping cost
- Other Variables
    - # of automobiles from each plant
    - # of automobiles to each region
- Constraints
    - # from each plant ≤ plant capacity
    - # to each region ≥ region demand

Example 2 (Assignment Model):

The city of Spring View is taking bids from six bus companies on the eight routes that must be driven in the surrounding school district. Each company enters a bid of how much it will charge to drive selected routes, although not all companies bid on all routes. The data are listed in Table 2. (If a company does not bid on a route, the corresponding entry is blank.) The city must decide which companies to assign to which routes with the specifications that (1) if a company does not bid on a route, it cannot be assigned to that route; (2) exactly one company must be assigned to each route; and (3) a company can be assigned to at most two routes. The objective is to minimize the total cost of covering all routes.

	R1	R2	R3	R4	R5	R6	R7	R8
C1		8200	7800	5400		3900		
C2	7800	8200		6300		3300	4900	
C3		4800				4400	5600	3600
C4			8000	5000	6800		6700	4200
C5	7200	6400		3900	6400	2800		3000
C6	7000	5800	7500	4500	5600		6000	4200

Table 2: Problem Data

Model Definitions:

- Input Variables
  - bids for routes
  - max # of bus routes per company
- Decision Variables
  - assignment of companies to bus routes
- Objective Function
  - total cost
- Other Variables
  - # of routes assigned to each company
  - # of companies assigned to each bus route
- Constraints
  - # of routes assigned ≤ max # of routes per company
  - # of companies assigned to each route = 1

Example 3 (Network Flows):

The RedBrand Company produces a tomato product at three plants. This product can be shipped directly to the company's two customers or it can first be shipped to the company's two warehouses and then to the customers. Figure 3 is a network representation of RedBrand's problem. Nodes 1, 2, and 3 represent the plants (these are the origins, denoted by S for supplier), nodes 4 and 5 represent the warehouses (these are the transshipment points, denoted by T), and nodes 6 and 7 represent the customers (these are the destinations, denoted by D). Note that some shipments are allowed among plants, among warehouses, and among customers. Also, some arcs have arrows on both ends. This means that flow is allowed in either direction.

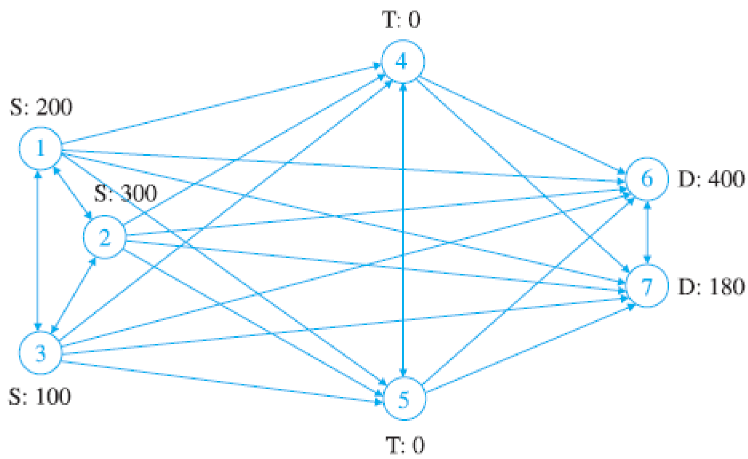

Figure 3: Network Representation of the Problem

The cost of producing the product is the same at each plant, so RedBrand is concerned with minimizing the total shipping cost incurred in meeting customer demands. The production capacity of each plant (in tons per year) and the demand of each customer are shown in Figure 3. For example, plant 1 (node 1) has a capacity of 200, and customer 1 (node 6) has a demand of 400. In addition, the cost (in thousands of dollars) of shipping a ton of the product between each pair of locations is listed in Table 3, where a blank indicates that RedBrand cannot ship along that arc. We also assume that at most 200 tons of the product can be shipped between any two nodes. This is the common arc capacity. RedBrand wants to determine a minimum-cost shipping schedule.

From/To	1	2	3	4	5	6	7
1		5.0	3.0	5.0	5.0	20.0	20.0
2	9.0		9.0	1.0	1.0	8.0	15.0
3	0.4	8.0		1.0	0.5	10.0	12.0
4					1.2	2.0	12.0
5				0.8		2.0	12.0
6							1.0
7						7.0	

Table 3: Problem Data

Model Definitions:

- Input Variables
    - plant capacities
    - customer demands
    - shipping cost
    - arc capacity
- Decision Variables
    - shipments
- Objective Function
    - total cost
- Other Variables
    - in-out flows
- Constraints
    - flows on arcs ≤ arc capacity
    - flow balances

Example 4 (Logistics Example):

Maude Jenkins, a 90-year-old woman, is planning to walk across the state, west to east, to gain support for a political cause she favors.6 She wants to travel the shortest distance to get from city 1 to city 10, using the arcs (roads) shown in Figure 4. The numbers on the arcs are miles. Arcs with double-headed arrows indicate that travel is possible in both directions (with the same distance in both directions). What route should Maude take?

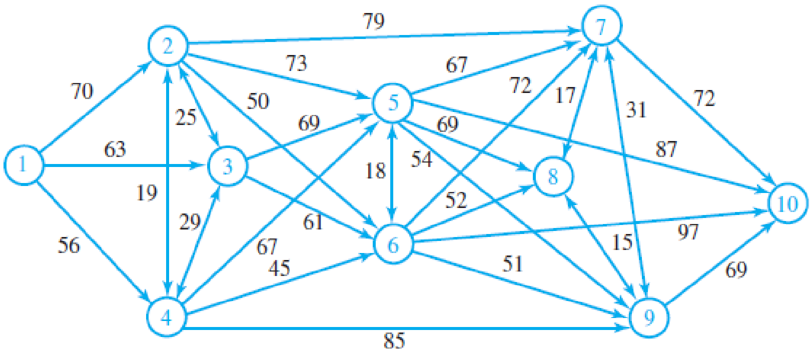

Figure 4: Network Representation of the Problem

Model Definitions:

- Input Variables
  - network structure
  - arc distances
- Decision Variables
  - arc flows
- Objective Function
  - total cost (distance)
- Other Variables
  - in-out flows
- Constraints
  - flow balances