## LP Examples*

## Example 1: Advertising Models

The General Flakes Company sells a brand of low-fat breakfast cereal that appeal to people of all age groups and both genders. The company advertises this cereal in a variety of 30-second television ads, and these ads can be placed in a variety of television shows. The ads in different shows vary by cost-some 30 -second slots are much more expensive than others - and by the types of viewers they are likely to reach. The company has segmented the potential viewers into six mutually exclusive categories: males age 18 to 35, males age 36 to 55, males over 55 , females age 18 to 35 , females age 36 to 55 , and females over 55. A rating service can supply data on the numbers of viewers in each of these categories who will watch a 30 -second ad on any particular television show. Each such viewer is called an exposure. The company has determined the required number of exposures it wants to obtain for each group. It wants to know how many ads to place on each of several television shows to obtain these required exposures at minimum cost. The data on costs per ad, numbers of exposures per ad, and minimal required exposures are listed in Table 1, where numbers of exposures are expressed in millions, and costs are in thousands of dollars. What should the company do?

|  |  |  |  |  |  |  | $\sum_{\lambda}^{2}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Men Category 1 | 5 | 6 | 5 | 0.5 | 0.7 | 0.1 | 0.1 | 3 | 60 |
| Men Category 2 | 3 | 5 | 2 | 0.5 | 0.2 | 0.1 | 0.2 | 5 | 60 |
| Men Category 3 | 1 | 3 | 0 | 0.3 | 0.0 | 0.0 | 0.3 | 4 | 28 |
| Women Category 1 | 6 | 1 | 4 | 0.1 | 0.9 | 0.6 | 0.1 | 3 | 60 |
| Women Category 2 | 4 | 1 | 2 | 0.1 | 0.1 | 1.3 | 0.2 | 5 | 60 |
| Women Category 3 | 2 | 1 | 0 | 0.0 | 0.0 | 0.4 | 0.3 | 4 | 28 |
| Cost per Ad | 140 | 100 | 80 | 9.0 | 13.0 | 15.0 | 8.0 | 140 |  |

Table 1: Model Data

[^0]We let

- $x_{i}$ be the number of ads purchased during TV show $i, i=1, \ldots, 8$
- $c_{i}$ be the cost of ads purchased during TV show $i, i=1, \ldots, 8$
- $a_{i j}$ be the number of exposures for category $j, j=1, \ldots, 6$ for TV show $i, i=1, \ldots, 8$
- $b_{j}$ be the min number of exposures needed for category $j$

$$
\begin{aligned}
\min z & =\sum_{i=1}^{8} c_{i} x_{i} \\
\sum_{j}^{6} a_{i j} x_{i} & \geq b_{j}, \quad \forall i \\
x_{i} & \geq 0, \quad \forall i, j
\end{aligned}
$$

## Example 2: Workforce Scheduling Models

A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required each day is given in Table 2. Union rules state that each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Monday to Friday must be off on Saturday and Sunday. The post office wants to meet its daily requirements using only full-time employees. Its objective is to minimize the number of full-time employees on its payroll.

| Day | Min \# of Employees Required |
| :--- | :---: |
| Monday | 17 |
| Tuesday | 13 |
| Wednesday | 15 |
| Thursday | 19 |
| Friday | 14 |
| Saturday | 16 |
| Sunday | 11 |

Table 2: Model Data

We let

- $\quad x_{i}$ be the \# of employees beginning work on day $i, i=1, \ldots, 7$


## Example: Aggregate Planning Models

During the next four months the SureStep Company must meet (on time) the following demands for pairs of shoes: 3000 in month 1; 5000 in month 2; 2000 in month 3; and 1000 in month 4. At the beginning of month 1, 500 pairs of shoes are on hand, and SureStep has 100 workers. A worker is paid $\$ 1500$ per month. Each worker can work up to 160 hours a month before he or she receives overtime. A worker can work up to 20 hours of overtime per month and is paid $\$ 13$ per hour for overtime labor. It takes four hours of labor and $\$ 15$ of raw material to produce a pair of shoes. At the beginning of each month, workers can be hired or fired. Each hired worker costs $\$ 1600$, and each fired worker costs $\$ 2000$. At the end of each month, a holding cost of $\$ 3$ per pair of shoes left in inventory is incurred. Production in a given month can be used to meet that month's demand. SureStep wants to use LP to determine its optimal production schedule and labor policy.

We let

- $x_{i}$ be the \# of workers hired for month $i, i=1, \ldots, 4$
- $y_{i}$ be the \# of workers fired for month $i, i=1, \ldots, 4$
- $t_{i}$ be the overtime used for month $i, i=1, \ldots, 4$
- $p_{i}$ be the production used for month $i, i=1, \ldots, 4$


## Example: Blending Models

Chandler Oil has 5000 barrels of crude oil 1 and 10,000 barrels of crude oil 2 available. Chandler sells gasoline and heating oil. These products are produced by blending the two crude oils together. Each barrel of crude oil 1 has a "quality level" of 10 and each barrel of crude oil 2 has a quality level of 5 . Gasoline must have an average quality level of at least 8 , whereas heating oil must have an average quality level of at least 6 . Gasoline sells for $\$ 75$ per barrel, and heating oil sells for $\$ 60$ per barrel. We assume that demand for heating oil and gasoline is unlimited, so that all of Chandler's production can be sold. Chandler wants to maximize its revenue from selling gasoline and heating oil.

## Example: Production Process Models

Repco produces three drugs, $\mathrm{A}, \mathrm{B}$, and C , and can sell these drugs in unlimited quantities at unit prices \$8, \$70, and \$100, respectively. Producing a unit of drug A requires one hour of labor. Producing a unit of drug $B$ requires two hours of labor and two units of drug A. Producing one unit of drug $C$ requires three hours of labor and one unit of drug $B$. Any drug $A$ that is used to produce drug $B$ cannot be sold separately, and any drug $B$ that is used to produce drug $C$ cannot be sold separately. A total of 4000 hours of labor are available. Repco wants to use LP to maximize its sales revenue.

## Example: Financial Models

At the present time, the beginning of year 1, the Barney-Jones Investment Corporation has $\$ 100,000$ to invest for the next four years. There are five possible investments, labeled A through E. The timing of cash outflows and cash inflows for these investments is somewhat irregular. For example, to take part in investment A, cash must be invested at the beginning of year 1, and for every dollar invested, there are returns of $\$ 0.50$ and $\$ 1.00$ at the beginnings of years 2 and 3. Information for the other investments follows, where all returns are per dollar invested:

- Investment B: Invest at the beginning of year 2, receive returns of $\$ 0.50$ and $\$ 1.00$ at the beginnings of years 3 and 4
- Investment C: Invest at the beginning of year 1, receive return of $\$ 1.20$ at the beginning of year 2
- Investment D: Invest at the beginning of year 4, receive return of $\$ 1.90$ at the beginning of year 5
- Investment E: Invest at the beginning of year 3, receive return of $\$ 1.50$ at the beginning of year 4

We assume that any amounts can be invested in these strategies and that the returns are the same for each dollar invested. However, to create a diversified portfolio, Barney-Jones wants to limit the amount put into any investment to $\$ 75,000$. The company wants an investment strategy that maximizes the amount of cash on hand at the beginning of year 5. At the beginning of any year, it can invest only cash on hand, which includes returns from previous investments. Any cash not invested in any year can be put in a short-term money market account that earns 3\% annually.

## Example: Data Envelopment Analysis

Consider a group of three hospitals. To keep the model small, assume that each hospital uses two inputs to produce three outputs. (In a real DEA, there are typically many more inputs and outputs.) The two inputs used by each hospital are

- I1: capital (measured by hundreds of hospital beds)
- I2: labor (measured by thousands of labor hours used in a month)

The outputs produced by each hospital are

- O1: hundreds of patient-days during month for patients under age 14
- O2: hundreds of patient-days during month for patients between 14 and 65
- O3: hundreds of patient-days for patients over 65

The inputs and outputs for these hospitals are given in Table 3. Which of these three hospitals is efficient in terms of using its inputs to produce outputs?

|  | Input 1 | Input 2 | Output 1 | Output 2 | Output 3 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Hospital 1 | 5 | 14 | 9 | 4 | 16 |
| Hospital 2 | 8 | 15 | 5 | 7 | 10 |
| Hospital 3 | 7 | 12 | 4 | 9 | 13 |

Table 3: Model Data


[^0]:    * Practical Management Science by Wayne L. Winston and S. Christian Albright

