END5504: Integer Programming

Example 1 (Fixed-Cost Models):

The Great Threads Company is capable of manufacturing shirts, shorts, pants, skirts, and jackets. Each type of clothing requires that Great Threads have the appropriate type of machinery available. The machinery needed to manufacture each type of clothing must be rented at the weekly rates shown in Table 1. This table also lists the amounts of cloth and labor required per unit of clothing, as well as the selling price and the unit variable cost for each type of clothing. In a given week, 4000 labor hours and 4500 square yards (sq. yd.) of cloth are available. The company wants to find a solution that maximizes its weekly profit. Develop a mathematical model to maximize the profit.

	Rental Cost	Labor Hours	Cloth	Selling Price	Variable Cost
Shirts	1500	2.0	3.0	35	20
Shorts	1200	1.0	2.5	40	10
Pants	1600	6.0	4.0	65	25
Skirts	1500	4.0	4.5	70	30
Jackets	1600	8.0	5.5	110	35

Table 1: Problem Data

Model Definitions:

- Input Variables
  - rental costs
  - resource usages
  - selling prices
  - variable costs
  - resource availabilities
- Decision Variables
  - whether to produce or not
  - how many to produce
- Objective Function
  - total profit
- Other Variables
  - resources used
  - upper limits on amounts to produce
  - total revenue
  - total variable cost
  - total fixed cost
- Constraints
  - amount produced ≤ capacity
  - resources used ≤ resources available

Example 2 (Either-Or Models):

Dorian Auto is considering manufacturing three types of cars (compact, midsize, and large) and two types of minivans (midsize and large). The resources required and the profit contributions yielded by each type of vehicle are shown in Table 2. At present, 6500 tons of steel and 65,000 hours of labor are available. If any vehicles of a given type are produced, production of that type of vehicle is economically feasible only if at least a minimal number of that type are produced. These minimal numbers are also listed in Table 2. Dorian wants to find a production schedule that maximizes its profit.

	Compact Car	Midsize Car	Large Car	Midsize Minivan	Large Minivan
Steel	1.5	3.0	5.0	6.0	8.0
Labor Hours	30.0	25.0	40.0	45.0	55.0
Min Production	1,000.0	1,000.0	1,000.0	200.0	200.0
Profit	2,000.0	2,500.0	3,000.0	5,500.0	7,000.0

Table 2: Problem Data

Model Definitions:

- Input Variables
    - resources
    - profits
    - min production capacity
    - resource availabilities
- Decision Variables
    - whether to produce or not
    - how many to produce
- Objective Function
    - profit
- Other Variables
    - lower and upper limits on amounts to produce
    - resources used
- Constraints
    - amount produced ≥ lower bounds
    - amount produced ≤ upper bounds
    - resources used ≤ resources available

Example 3 (Set-Covering Models):

Western Airlines wants to design a hub system in the United States. Each hub is used for connecting flights to and from cities within 1000 miles of the hub. Western runs flights among the following cities: Atlanta, Boston, Chicago, Denver, Houston, Los Angeles, New Orleans, New York, Pittsburgh, Salt Lake City, San Francisco, and Seattle. The company wants to determine the smallest number of hubs it needs to cover all these cities, where a city is covered if it is within 1000 miles of at least one hub. Table 3 lists which cities are within 1,000 miles of other cities. Develop a binary model to find the minimum number of hub locations that can cover all cities.

City	Cities within 1,000 miles
Atlanta (AT)	AT, CH, HO, NO, NY, PI
Boston (BO)	BO, NY, PI
Chicago (CH)	AT, CH, NY, NO, PI
Denver (DE)	DE, SL
Houston (HO)	AT, HO, NO
Los Angeles (LA)	LA, SL, SF
New Orleans (NO)	AT, CH, HO, NO
New York (NY)	AT, BO, CH, NY, PI
Pittsburgh (PI)	AT, BO, CH, NY, PI
Salt Lake City (SL)	DE, LA, SL, SF, SE
San Francisco (SF)	LA, SL, SF, SE
Seattle (SE)	SL, SF, SE

Table 3: Problem Data

Model Definitions:

- Input Variables
    - cities within 1,000 miles
- Decision Variables
    - hub locations (binary)
- Objective Function
    - # of hubs
- Other Variables
    - # of hubs covering each city
- Constraints
    - # of hubs covering each city ≥ 1

Example 3 (Cutting Stock Models):

The Rheem Paper Company produces rolls of paper of various types for its customers. One type is produced in standard rolls that are 60 inches wide and (when unwound) 200 yards long. Customers for this type of paper order rolls that are all 200 yards long, but can have any of the widths 12, 15, 20, 24, 30, or 40 inches. In a given week, Rheem waits for all orders and then decides how to cut its 60-inch rolls to satisfy the orders. For example, if there are five orders for 15-inch widths and two orders for 40-inch widths, Rheem could satisfy the order by producing three rolls, cutting each of the first two into a 40-inch and a 15-inch cut (with 5 inches left over) and cutting the third into four 15-inch cuts (with one of these left over). Each week, Rheem must decide how to cut its rolls in the most economical way to meet its orders. Specifically, it wants to cut as few rolls as possible. The objective is to find a way of cutting paper rolls in various widths so as to satisfy all customer orders and minimize the total number of rolls cut.

Model Definitions:

- Input Variables
    - width of roll
    - number of rolls of possible widths required
    - list of patterns
- Decision Variables
    - # of rolls cut for each pattern
- Objective Function
    - # of rolls cut total
- Other Variables
    - # of each width obtained
- Constraints
    - # of each width obtained ≥ # of each width required