# Introduction to Constraint Programming

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# Constraint Programming

- Formulate the model using a variety of constraint types.
- Find the feasible solutions that satisfy all constraints.
- Search for the optimal solution.

# Constraint Programming

Example: Suppose that we have the four variables, $x_1, x_2, x_3, x_4$, with their domains

$$x_1 \in \{1,2\}$$

$$x_2 \in \{1,2\}$$

$$x_3 \in \{1,2,3\}$$

$$x_4 \in \{1,2,3,4,5\}$$

# Constraint Programming

We also have the following constraints.

$$x_i \neq x_j, \quad i \neq j$$

$$x_1 + x_3 = 4$$

# Domain Reduction & Constraint Propagation

Since $x_1 \in \{1,2\}$ and $x_2 \in \{1,2\}$, the first constraint $x_i \neq x_j, \quad i \neq j$

implies that

$$x_3 \in \{3\}$$

It then implies again

$$x_4 \in \{4,5\}$$

# Domain Reduction & Constraint Propagation

We can then write,

$$x_1 \in \{1\}$$

$$x_2 \in \{2\}$$

$$x_3 \in \{3\}$$

$$x_4 \in \{4,5\}$$

# Example Constraints

The “All-Different” Constraint

$$\text{all-different}\ (y_1, y_2, \dots, y_n)$$

The “Element” Constraint

$$\text{element}\ (y, [c_1, c_2, \dots, c_n], z)$$

# Assignment Problem

$$\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \forall j$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \forall i$$

$$x_{ij} \in \{0,1\}$$

# Assignment Problem

If we define $y_i$ as the task assigned to person $i$, we can write

$$\min z = \sum_{i=1}^{n} z_i$$

$$\text{element } (y_i, [c_{i1}, c_{i2}, \dots, c_{in}], z_i), \qquad i = 1, \dots, n$$

$$\text{all-different } (y_1, y_2, \dots, y_n)$$

$$y_i \in \{1, \dots, n\}, \quad i = 1, \dots, n$$

# IBM ILOG CPLEX CP Optimizer

```
using CP;

int nbPerm = ...;
range r = 1..nbPerm;
int dist[r][r] = ...;
int flow[r][r] = ...;

execute{
      cp.param.timeLimit=30;
}

dvar int perm[1..nbPerm] in r;

dexpr int cost[i in r][j in r] =
dist[i][j]*flow[perm[i]][perm[j]];

minimize sum(i in r, j in r) cost[i][j];
subject to {
      allDifferent(perm);
};
```

# The End

Questions?