System Simulation
Chapter 2: Simulation Examples

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March 29, 2014

Introduction

- Several examples of simulation that can be performed by devising a simulation table either manually or with a spreadsheet.
- The simulation table provides a systematic method for tracking system state over time.
- These examples provide insight into the methodology of discrete-system simulation and the descriptive statistics used for predicting system performance.

Introduction - cont.

The simulations in this chapter are carried out by following three steps:

1. Determine the characteristics of each of the inputs to the simulation. Quite often, these are modeled as (discrete - continuous) probability distributions.
2. Construct a simulation table. Each table is different based on the problem of interest. In the example simulation table in your text there are p inputs, x_{ij}, $j = 1, 2, ..., p$, and one response, y_i for each repetition i, $i = 1, 2, ..., n$.
3. For each repetition i, generate a value for each of the p inputs, and evaluate the function calculating a value of the response y_i.

Introduction - cont.

The simulations examples in this chapter are in

- queuing (two examples with one and two servers)
- inventory
- reliability
- network analysis

Introduction

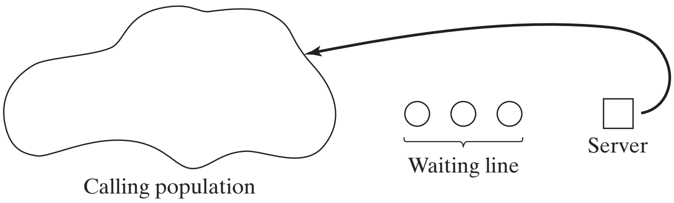

Figure : Single-Server Queuing System

Introduction - cont.

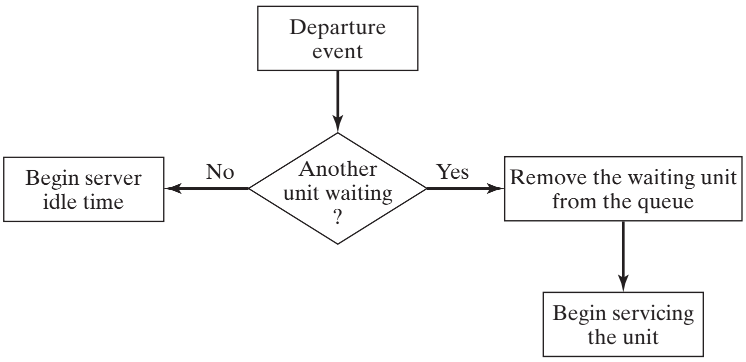

Figure : Departure Event

Introduction - cont.

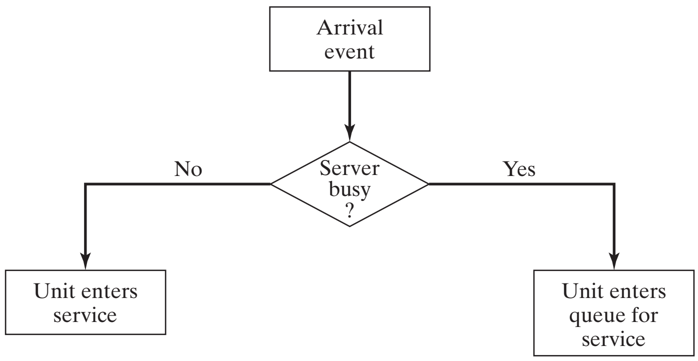

Figure : Arrival Event

Introduction - cont.

		Queue status	
		Not empty	Empty
Server status	Busy	Enter queue	Enter queue
	Idle	Impossible	Enter service

Figure : System State upon Arrival

Introduction - cont.

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

Figure : System State upon Departure

Single-Channel Queue

- Inputs
  - Time Between Arrivals
  - Service Time
- Outputs (Performance Measures)
  - Average Number in Queue
  - Average Waiting Time
  - What else?

Single-Channel Queue

Table : Distribution of Time Between Arrivals

Time Between Arrivals	Probability	Cumulative Probability	Random Digits
1	0.125	0.125	001 - 125
2	0.125	0.250	126 - 250
3	0.125	0.375	251 - 375
4	0.125	0.500	376 - 500
5	0.125	0.625	501 - 625
6	0.125	0.750	626 - 750
7	0.125	0.875	751 - 875
8	0.125	1.000	876 - 000

Single-Channel Queue - cont.

Table : Service-Time Distribution

Service Time	Probability	Cumulative Probability	Random Digits
1	0.10	0.10	01 - 10
2	0.20	0.30	11 - 30
3	0.30	0.60	31 - 60
4	0.25	0.85	61 - 85
5	0.10	0.95	86 - 95
6	0.05	1.00	96 - 00

Single-Channel Queue - cont.

	A	B	C	D	E	F	G	H	I	J
15		TOTALS	420		320		163		483	106
16		AVERAGES	4.24		3.20		1.63		4.83	1.07
17		Number of Customers=			100					
18		Simulation Table								
19		Step	Activity	Clock	Activity	Clock	Output	Clock	Output	Output
20–22		Customer	Interarrival Time (Minutes)	Arrival Time	Service Time (Minutes)	Time Service Begins	Waiting Time in Queue (Minutes)	Time Service Ends	Time Customer Spends in System (Minutes)	Idle Time of Server (Minutes)
23		1	0	0	2	0	0	2	2	
24		2	5	5	2	5	0	7	2	3
25		3	5	10	4	10	0	14	4	3
26		4	4	14	4	14	0	18	4	0
27		5	2	16	3	18	2	21	5	0
28		6	8	24	2	24	0	26	2	3
29		7	7	31	3	31	0	34	3	5
30		8	8	39	5	39	0	44	5	5
31		9	5	44	1	44	0	45	1	0
32		10	2	46	6	46	0	52	6	1
33		11	1	47	4	52	5	56	9	0

Figure : Simulation Table for Single-Channel Queuing Problem

Single-Channel Queue - cont.

Some of the findings from our simulation model:

- average waiting time, the probability that a customer has to wait

$$\begin{aligned}\text{average waiting time} &= \frac{\text{total time customers wait in queue}}{\text{total number of customers}}\\ &= \frac{174}{100}\\ &= 1.74\end{aligned}$$

$$\begin{aligned}\text{probability (wait)} &= \frac{\text{number of customers who wait in queue}}{\text{total number of customers}}\\ &= \frac{46}{100}\\ &= 0.46\end{aligned}$$

Single-Channel Queue - cont.

- proportion of idle - busy time of the server

$$\begin{aligned} \text{probability (server idle)} &= \frac{\text{total idle time of server}}{\text{total run time}} \\ &= \frac{101}{418} \\ &= 0.24 \end{aligned}$$

$$\begin{aligned} \text{probability (server busy)} &= 1 - \text{probability (server idle)} \\ &= 1 - 0.24 \\ &= 0.76 \end{aligned}$$

Single-Channel Queue - cont.

- average versus expected service times

$$\begin{aligned} \text{average service time} &= \frac{\text{total service time}}{\text{total number of customers}} \\ &= \frac{317}{100} \\ &= 3.17 \end{aligned}$$

$$E(S) = \sum_{s=0}^{\infty} sp(s) = 1(.10) + 2(.20) + \ldots + 6(.05) = 3.2$$

- They will get closer to each other if we simulate longer.

Single-Channel Queue - cont.

- average versus expected time between arrivals

$$\begin{aligned}\text{average time between arrivals} &= \frac{\text{sum of all times between arrivals}}{\text{number of arrivals} - 1} \\ &= \frac{415}{99} \\ &= 4.19\end{aligned}$$

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5$$

- Again, they will get closer to each other if we simulate longer.

Single-Channel Queue - cont.

- average waiting time of waiting customers

$$\begin{aligned}\text{average waiting time of waiting customers} &= \frac{\text{total time customers wait in queue}}{\text{total number of waiting customers}} \\ &= \frac{174}{54} \\ &= 3.22\end{aligned}$$

Single-Channel Queue - cont.

- average time customer spends in system

$$\begin{aligned}\text{average time in system} &= \frac{\text{total time in system}}{\text{total number of customers}} \\ &= \frac{491}{100} \\ &= 4.91\end{aligned}$$

- It can also be computed as

$$\begin{aligned}\begin{array}{c}\text{average time}\\ \text{in system}\end{array} &= \begin{array}{c}\text{average time}\\ \text{in queue}\end{array} + \begin{array}{c}\text{average time}\\ \text{in service}\end{array} \\ &= 1.74 + 3.17 \\ &= 4.91\end{aligned}$$

Able-Baker Call Center Problem

- Inputs
  - Time Between Calls
  - Service Time for Able
  - Service Time for Baker
- Outputs (Performance Measures)
  - Caller Delay
  - What else?

Able-Baker Call Center Problem

	B	C	D
4	Interarrival Distribution of Calls		
5–7	**Interarrival Time (Minutes)**	**Probability**	**Cumulative Probability**
8	1	0.25	0.25
9	2	0.40	0.65
10	3	0.20	0.85
11	4	0.15	1.00

Figure : Distribution of Time Between Arrivals

Able-Baker Call Center Problem - cont.

	F	G	H
4	Able's Service Time Distribution		
5–7	**Service Times (Minutes)**	**Probability**	**Cumulative Probability**
8	2	0.30	0.30
9	3	0.28	0.58
10	4	0.25	0.83
11	5	0.17	1.00

Figure : Frequency Distribution of Average Waiting Times

Able-Baker Call Center Problem - cont.

	I	J	K
4	Baker's Service Time Distribution		
5-7	Service Times (Minutes)	Probability	Cumulative Probability
8	3	0.35	0.35
9	4	0.25	0.60
10	5	0.20	0.80
11	6	0.20	1.00

Figure : Frequency Distribution of Average Waiting Times

Able-Baker Call Center Problem - cont.

	B	C	D	E	F	G	H	I	J	K	L	M
13	TOTALS										73	432
14		Number of Callers=		100		Seed for Random Numbers			12,345			
15	Simulation Table											
16	Step	Activity	Clock	Clock	Clock	State	Activity	Clock	Clock	Clock	Output	Output
17-18									Service Completion Time			
19	Caller Number	Interarrival Time (Minutes)	Arrival Time	When Able Available	When Baker Available	Server Chosen	Service Time (Minutes)	Time Service Begins	Able	Baker	Caller Delay (Minutes)	Time in System (Minutes)
20	1		0	0	0	Able	4	0	4		0	4
21	2	1	1	4	0	Baker	4	1		5	0	4
22	3	1	2	4	5	Able	3	4	7		2	5
23	4	3	5	7	5	Baker	3	5		8	0	3
24	5	2	7	7	8	Able	5	7	12		0	5
25	6	2	9	12	8	Baker	5	9		14	0	5
26	7	1	10	12	14	Able	5	12	17		2	7
27	8	2	12	17	14	Baker	4	14		18	2	6
28	9	4	16	17	18	Able	2	17	19		1	3
29	10	1	17	19	18	Baker	3	18		21	1	4

Figure : Simulation Table

Able-Baker Call Center Problem - cont.

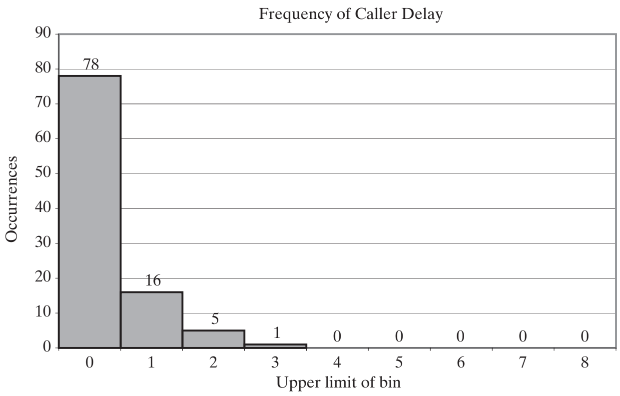

Figure : Caller Delay for First Trial

Able-Baker Call Center Problem - cont.

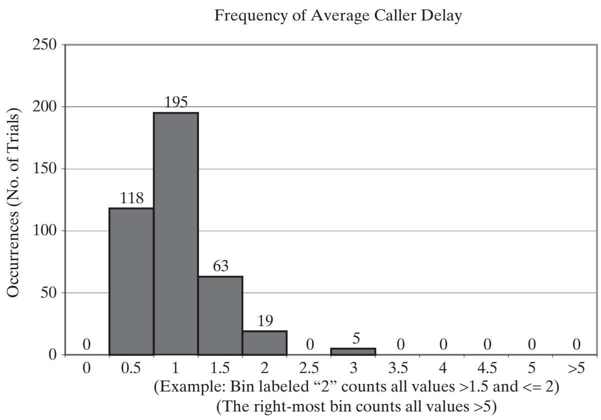

Figure : Caller Delay for 400 Trials

Introduction

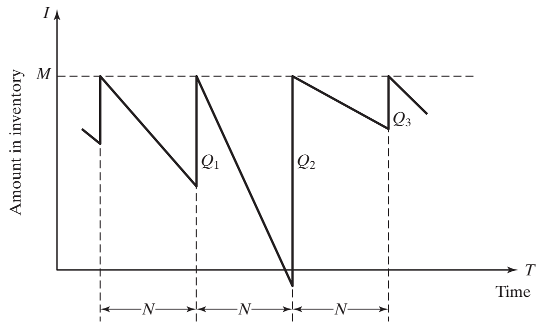

Figure : Overview

Newsboy Problem

- Inputs
  - Day Type Distribution
  - Demand Distribution
- Outputs (Performance Measures)
  - Total Profit
  - What else?

Newsboy Problem

Table : Newspaper Demand Distribution

Demand	Good	Fair	Poor
40	0.03	0.10	0.44
50	0.05	0.18	0.22
60	0.15	0.40	0.16
70	0.20	0.20	0.12
80	0.35	0.08	0.06
90	0.15	0.04	0.00
100	0.07	0.00	0.00

Newsboy Problem - cont.

	J	K	L
4	Type of Newsday		
5 6	Type	Probability	Cumulative Probability
7	Good	0.35	0.35
8	Fair	0.45	0.80
9	Poor	0.20	1.00

Figure : Random Digits for Type of Newsday

Newsboy Problem - cont.

	B	C	D	E	F	G	H
4	**Distribution of Newspapers Demanded**						
5	**Demand**	**Demand Probabilities**			**Cumulative Probabilities**		
6		**Good**	**Fair**	**Poor**	**Good**	**Fair**	**Poor**
7	40	0.03	0.10	0.44	0.03	0.10	0.44
8	50	0.05	0.18	0.22	0.08	0.28	0.66
9	60	0.15	0.40	0.16	0.23	0.68	0.82
10	70	0.20	0.20	0.12	0.43	0.88	0.94
11	80	0.35	0.08	0.06	0.78	0.96	1.00
12	90	0.15	0.04	0.00	0.93	1.00	1.00
13	100	0.07	0.00	0.00	1.00	1.00	1.00

Figure : Random Digits for Newspaper Demand

Newsboy Problem - cont.

$$\text{profit} = \begin{pmatrix} \text{revenue} \\ \text{from} \\ \text{sales} \end{pmatrix} - \begin{pmatrix} \text{cost} \\ \text{of} \\ \text{papers} \end{pmatrix} - \begin{pmatrix} \text{lost profit} \\ \text{from} \\ \text{excess demand} \end{pmatrix} + \begin{pmatrix} \text{salvage} \\ \text{from} \\ \text{scrap} \end{pmatrix}$$

Newsboy Problem - cont.

Simulate for 20 days where, for each newspaper,

- the cost is 33 cents
- the revenue from sales is 50 cents
- the lost profit from excess demand is 17 cents
- the salvage value is 5 cents

Newsboy Problem - cont.

	B	C	D	E	F	G	HI	I
16	Simulation Table							
17, 18, 19	Day	Type of Newsday	Demand	Revenue from Sales	Lost Profit from Excess Demand	Salvage from Sale of Scrap	Daily Cost	Daily Profit
20	1	Fair	50	$25.00	$0.00	$1.00	$23.10	$2.90
21	2	Fair	50	$25.00	$0.00	$1.00	$23.10	$2.90
22	3	Fair	70	$35.00	$0.00	$0.00	$23.10	$11.90
23	4	Good	80	$35.00	$1.70	$0.00	$23.10	$10.20
24	5	Poor	40	$20.00	$0.00	$1.50	$23.10	-$1.60
25	6	Fair	50	$25.00	$0.00	$1.00	$23.10	$2.90
26	7	Poor	50	$25.00	$0.00	$1.00	$23.10	$2.90
27	8	Fair	70	$35.00	$0.00	$0.00	$23.10	$11.90
28	9	Good	40	$20.00	$0.00	$1.50	$23.10	-$1.60
29	10	Good	100	$35.00	$5.10	$0.00	$23.10	$6.80
30	11	Fair	70	$35.00	$0.00	$0.00	$23.10	$11.90
31	12	Poor	50	$25.00	$0.00	$1.00	$23.10	$2.90
32	13	Fair	50	$25.00	$0.00	$1.00	$23.10	$2.90
33	14	Poor	40	$20.00	$0.00	$1.50	$23.10	-$1.60
34	15	Good	80	$35.00	$1.70	$0.00	$23.10	$10.20
35	16	Good	70	$35.00	$0.00	$0.00	$23.10	$11.90
36	17	Good	80	$35.00	$1.70	$0.00	$23.10	$10.20
37	18	Poor	40	$20.00	$0.00	$1.50	$23.10	-$1.60
38	19	Fair	70	$35.00	$0.00	$0.00	$23.10	$11.90
39	20	Good	100	$35.00	$5.10	$0.00	$23.10	$6.80
40							TOTAL PROFIT =	$114.70

Figure : Simulation Table

Newsboy Problem - cont.

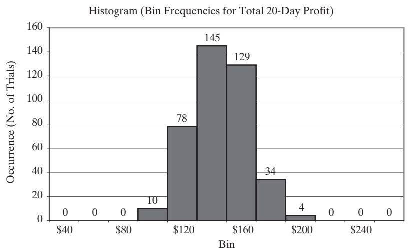

Figure : Total Profit

Newsboy Problem - cont.

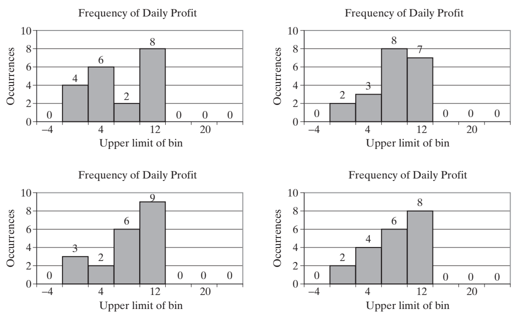

Figure : Daily Profits

Order Up-To-Level Inventory System

- Inputs
  - Little m
  - Big M
  - Demand Distribution
  - Lead Time
- Outputs (Performance Measures)
  - Total Cost
  - Average Inventory Level
  - What else?

Order Up-To-Level Inventory System

	B	C	D
4	**Distribution of Daily Demand**		
5	**Demand**	**Probability**	**Cumulative**
6			**Probability**
7	0	0.10	0.10
8	1	0.25	0.35
9	2	0.35	0.70
10	3	0.21	0.91
11	4	0.09	1.00

Figure : Demand

Order Up-To-Level Inventory System - cont.

	F	G	H
4	Distribution of Lead Time		
5	Lead Time	Probability	Cumulative
6	(days)		Probability
7	1	0.60	0.60
8	2	0.30	0.90
9	3	0.10	1.00

Figure : Lead Time

Order Up-To-Level Inventory System - cont.

	B	C	D	E	F	G	H	I	J	K
13	Simulation Table									
14	Step	State	State	State	Input	State	State	State	Activity	State
15	Clock							Pending	Lead	Days until
16			Day within	Beginning		Ending	Shortage	Order	Time	Order
17	Day	Cycle	Cycle	Inventory	Demand	Inventory	Quantity	(Quantity)	(days)	Arrives
18	0	0	5	-	-	3	0	8	2	2
19	1	1	1	3	2	1	0	8		1
20	2	1	2	1	1	0	0			
21	3	1	3	8	2	6	0			
22	4	1	4	6	1	5	0			
23	5	1	5	5	2	3	0	8	1	1
24	6	2	1	3	3	0	0			
25	7	2	2	8	2	6	0			
26	8	2	3	6	3	3	0			
27	9	2	4	3	2	1	0			
28	10	2	5	1	3	0	2	13	2	2
29	11	3	1	0	1	0	3	13		1
30	12	3	2	0	2	0	5			
31	13	3	3	13	2	6	0			
32	14	3	4	6	3	3	0			
33	15	3	5	3	1	2	0	9	1	1
34	16	4	1	2	0	2	0			
35	17	4	2	11	4	7	0			
36	18	4	3	7	2	5	0			
37	19	4	4	5	3	2	0			
38	20	4	5	2	3	0	1	12	1	1
39	21	5	1	0	2	0	3			
40	22	5	2	12	1	8	0			
41	23	5	3	8	4	4	0			
42	24	5	4	4	1	3	0			
43	25	5	5	3	1	2	0	9	1	1
44	TOTAL					69	14			
45	AVERAGE				2.04	2.76	0.56			

Figure : Simulation Table

Other Examples

- A Reliability problem (Example 2.5)
- Random Normal Numbers (Example 2.6)
- Lead Time Demand (Example 2.7)
- Project Simulation (Example 2.8)

A Reliability Problem

- Inputs
  - Bearing Life
  - Delay Time
- Outputs (Performance Measures)
  - Total Cost

A Reliability Problem

	B	C	D
3	Distribution of Bearing-Life		
4 5	Bearing Life	Probability	Cumulative Probability
6	1000	0.10	0.1
7	1100	0.13	0.23
8	1200	0.25	0.48
9	1300	0.13	0.61
10	1400	0.09	0.70
11	1500	0.12	0.82
12	1600	0.02	0.84
13	1700	0.06	0.90
14	1800	0.05	0.95
15	1900	0.05	1.00

Figure : Bearing Life Distribution

A Reliability Problem-cont.

	F	G	H
4	Distribution of Delay Time		
5 6	Delay Time	Probability	Cumulative Probability
7	5	0.60	0.60
8	10	0.30	0.90
9	15	0.10	1.00

Figure : Delay Time Distribution

A Reliability Problem

- When a break-down occurs, we call the repairman to fix it.
- Downtime cost for the mill is $10 per minute.
- Cost of the repairman is $30 per hour.
- It takes 20 minutes to replace 1 bearing, 30 minutes to replace 2 bearings and 40 minutes to replace 3 bearings.
- Current Method: We replace a bearing whenever it breaks-down.
- Proposed Method: We replace “all bearings” whenever one of them breaks-down.
- Which one is better?

A Reliability Problem-cont.

	B	C	D	E	F	G	H
17	Simulation Table						
18		Bearing 1		Bearing 2		Bearing3	
19		Life	Delay	Life	Delay	Life	Delay
20	Step	(Hours)	(minutes)	(Hours)	(minutes)	(Hours)	(minutes)
21	1	1000	5	1700	10	1300	10
22	2	1200	5	1100	5	1100	5
23	3	1200	10	1000	10	1300	10
24	4	1500	5	1000	10	1100	15
25	5	1700	5	1900	15	1200	5
26	6	1200	5	1200	10	1500	10
27	7	1300	5	1500	5	1100	10
28	8	1700	5	1700	5	1400	15
29	9	1000	5	1300	5	1800	15
30	10	1800	10	1300	5	1200	5
31	11	1200	5	1100	5	1500	5
32	12	1100	5	1800	5	1100	10
33	13	1300	10	1200	5	1700	10
34	14	1300	10	1100	5	1300	10
35	15	1100	5	1300	5	1300	10
36	TOTALS	19,600	95	20,200	105	19,900	145

Figure : Simulation Table-Current Method

A Reliability Problem-cont.

For 15 bearing changes, we have

- Cost of bearings $= 45 \times 32 = \$1,440$
- Cost of delay time $= (110 + 110 + 105) \times 10 = \$3,250$
- Cost of downtime $= 45 \times 20 \times 10 = \$9,000$
- Cost of repairman $= 45 \times 20 \times 30/60 = \$450$
- Total cost $= 1,440 + 3,250 + 9,000 + 450 = \$14,140$.

A Reliability Problem-cont.

	B	C	D	E	F	G
17	Simulation Table					
18		Bearing 1	Bearing 2	Bearing 3	First	
19		Life	Life	Life	Failure	Delay
20	Step	(Hours)	(Hours)	(Hours)	(Hours)	(minutes)
21	1	1300	1100	1300	1,100	10
22	2	1100	1200	1500	1,100	5
23	3	1100	1400	1800	1,100	15
24	4	1200	1500	1100	1,100	10
25	5	1700	1300	1300	1,300	5
26	6	1700	1100	1000	1,000	10
27	7	1000	1900	1200	1,000	10
28	8	1500	1700	1300	1,300	15
29	9	1300	1100	1800	1,100	5
30	10	1200	1100	1300	1,100	5
31	11	1000	1200	1200	1,000	15
32	12	1500	1700	1200	1,200	10
33	13	1300	1700	1000	1,000	10
34	14	1800	1200	1100	1,100	10
35	15	1300	1300	1100	1,100	10
36	TOTALS				16600	145

Figure : Simulation Table-Proposed Method

A Reliability Problem-cont.

For 15 bearing changes, we have

- Cost of bearings $= 45 \times 32 = \$1,440$
- Cost of delay time $= 110 \times 10 = \$1,100$
- Cost of downtime $= 15 \times 40 \times 10 = \$6,000$
- Cost of repairman $= 15 \times 40 \times 30/60 = \$300$
- Total cost $= 1,440 + 1,100 + 6,000 + 300 = \$8,840$.

Random Normal Numbers

Simulate a bombin operation as follows:

- If a bomb falls anywhere on target it is hit, otherwise it is a miss.
- The aiming point is $(0, 0)$.
- The point is impact is normally distributed around the aiming point with a standard deviation of 400 and 200 meters in x and y directions, respectively.
- Simulate the operation for 10 bombs.

Random Normal Numbers-cont.

Since

$$Z = \frac{X - \mu_X}{\sigma_X}, \quad Z = \frac{Y - \mu_Y}{\sigma_Y}$$

we can write

$$X = \mu_X + Z\sigma_X = 0 + 400Z, \quad Y = \mu_Y + Z\sigma_Y = 0 + 200Z$$

Random Normal Numbers

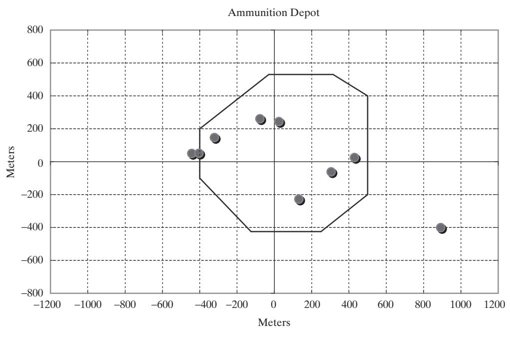

Figure : Simulated Bombing Run

Random Normal Numbers-cont.

	F	G	H	I	J	K	L	M	N	O	P	Q
3		**Point of Impact of Each Bomb(1-10)**										
4	**Bomb**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
5	**X**	891.8	-1257.3	429.6	24.5	-321.0	-77.3	131.4	304.5	-442.7	-403.9	**Number**
6	**Y**	-400.7	-159.4	25.3	243.6	146.5	260.7	-228.3	-62.0	49.7	51.0	**Hits**
7	**Hit ?**	Miss	Miss	Hit	Hit	Hit	Hit	Hit	Hit	Miss	Miss	6

Figure : Simulated Bombing Run

Summary

- Reading HW: Chapter 2.
- Chapter 2 Exercises